Machine Learning - Lecture 11

Deconstructing Decision Trees
(Randomized Trees, Forests, and Ferns)

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Course Outline

- **Fundamentals (2 weeks)**
  - Bayes Decision Theory
  - Probability Density Estimation

- **Discriminative Approaches (5 weeks)**
  - Linear Discriminant Functions
  - Statistical Learning Theory & SVMs
  - Ensemble Methods & Boosting
  - Randomized Trees, Forests & Ferns

- **Generative Models (4 weeks)**
  - Bayesian Networks
  - Markov Random Fields
Recap: Decision Trees

• Example:
  ➢ “Classify Saturday mornings according to whether they’re suitable for playing tennis.”

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Recap: CART Framework

Six general questions

1. Binary or multi-valued problem?
   - I.e. how many splits should there be at each node?

2. Which property should be tested at a node?
   - I.e. how to select the query attribute?

3. When should a node be declared a leaf?
   - I.e. when to stop growing the tree?

4. How can a grown tree be simplified or pruned?
   - Goal: reduce overfitting.

5. How to deal with impure nodes?
   - I.e. when the data itself is ambiguous.

6. How should missing attributes be handled?
Recap: Picking a Good Splitting Feature

• Goal
  - Select the query (=split) that decreases impurity the most
  \[ \Delta i(N) = i(N) - P_L i(N_L) - (1 - P_L) i(N_R) \]

• Impurity measures
  - Entropy impurity (information gain):
    \[ i(N) = - \sum_j p(C_j|N) \log_2 p(C_j|N) \]
  - Gini impurity:
    \[ i(N) = \sum_{i \neq j} p(C_i|N)p(C_j|N) = \frac{1}{2} \left[ 1 - \sum_j p^2(C_j|N) \right] \]
Recap: Overfitting Prevention (Pruning)

- Two basic approaches for decision trees
  - Prepruning: Stop growing tree as some point during top-down construction when there is no longer sufficient data to make reliable decisions.
    - Cross-validation
    - Chi-square test
    - MDL
  - Postpruning: Grow the full tree, then remove subtrees that do not have sufficient evidence.
    - Merging nodes
    - Rule-based pruning

- In practice often preferable to apply post-pruning.
Recap: Computational Complexity

- **Given**
  - Data points \( \{x_1, \ldots, x_N\} \)
  - Dimensionality \( D \)

- **Complexity**
  - Storage: \( O(N) \)
  - Test runtime: \( O(\log N) \)
  - Training runtime: \( O(DN^2 \log N) \)
    - Most expensive part.
    - Critical step: selecting the optimal splitting point.
    - Need to check \( D \) dimensions, for each need to sort \( N \) data points. \( O(DN \log N) \)
Summary: Decision Trees

• Properties
  - Simple learning procedure, fast evaluation.
  - Can be applied to metric, nominal, or mixed data.
  - Often yield interpretable results.
Summary: Decision Trees

• Limitations
  - Often produce noisy (bushy) or weak (stunted) classifiers.
  - Do not generalize too well.
  - Training data fragmentation:
    - As tree progresses, splits are selected based on less and less data.
  - Overtraining and undertraining:
    - Deep trees: fit the training data well, will not generalize well to new test data.
    - Shallow trees: not sufficiently refined.
  - Stability
    - Trees can be very sensitive to details of the training points.
    - If a single data point is only slightly shifted, a radically different tree may come out!
    => Result of discrete and greedy learning procedure.
  - Expensive learning step
    - Mostly due to costly selection of optimal split.
Topics of This Lecture

- **Randomized Decision Trees**
  - Randomized attribute selection

- **Recap: Random Forests**
  - Bootstrap sampling
  - Ensemble of randomized trees
  - Posterior sum combination
  - Analysis

- **Extremely randomized trees**
  - Random attribute selection

- **Ferns**
  - Fern structure
  - Semi-Naïve Bayes combination
  - Applications
Randomized Decision Trees (Amit & Geman 1997)

- Decision trees: main effort on finding good split
  - Training runtime: $O(DN^2 \log N)$
  - This is what takes most effort in practice.
  - Especially cumbersome with many attributes (large $D$).

- Idea: randomize attribute selection
  - No longer look for globally optimal split.
  - Instead randomly use subset of $K$ attributes on which to base the split.
  - Choose best splitting attribute e.g. by maximizing the information gain ($\triangle E$, reducing entropy):

$$
\Delta E = \sum_{k=1}^{K} \frac{|S_k|}{|S|} \sum_{j=1}^{N} p_j \log_2(p_j)
$$

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Randomized Decision Trees

- Randomized splitting
  - Faster training: $O(KN^2 \log N)$ with $K \ll D$.
  - Use very simple binary feature tests.
  - Typical choice
    - $K = 10$ for root node.
    - $K = 100d$ for node at level $d$.

- Effect of random split
  - Of course, the tree is no longer as powerful as a single classifier...
  - But we can compensate by building several trees.
Ensemble Combination

- Ensemble combination
  - Tree leaves \((l, \eta)\) store posterior probabilities of the target classes.
  - Combine the output of several trees by averaging their posteriors (Bayesian model combination)

\[
p(C|\mathbf{x}) = \frac{1}{L} \sum_{l=1}^{L} p_{l,\eta}(C|\mathbf{x})
\]

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Applications: Character Recognition

- **Computer Vision: Optical character recognition**
  - Classify small (14x20) images of hand-written characters/digits into one of 10 or 26 classes.

- **Simple binary features**
  - Tests for individual binary pixel values.
  - Organized in randomized tree.


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Applications: Character Recognition

- **Image patches (“Tags”)**
  - Randomly sampled 4×4 patches
  - Construct a randomized tree based on binary single-pixel tests
  - Each leaf node corresponds to a “patch class” and produces a tag

- **Representation of digits (“Queries”)**
  - Specific spatial arrangements of tags
  - An image answers “yes” if any such structure is found anywhere
  - How do we know which spatial arrangements to look for?
Applications: Character Recognition

- **Answer:** Create a second-level decision tree!
  - Start with two tags connected by an arc
  - Search through extensions of confirmed queries (or rather through a subset of them, there are lots!)
  - Select query with best information gain
  - Recurse...

- **Classification**
  - Average estimated posterior distributions stored in the leaves.

![Decision Tree Diagram]

Slide adapted from Jan Hosang
Applications: Fast Keypoint Detection

- **Computer Vision: fast keypoint detection**
  - Detect keypoints: small patches in the image used for matching
  - Classify into one of ~200 categories (visual words)

- **Extremely simple features**
  - E.g. pixel value in a color channel (CIELab)
  - E.g. sum of two points in the patch
  - E.g. difference of two points in the patch
  - E.g. absolute difference of two points

- **Create forest of randomized decision trees**
  - Each leaf node contains probability distribution over 200 classes
  - Can be updated and re-normalized incrementally.
Application: Fast Keypoint Detection

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  - Posterior sum combination
  - Analysis
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  - Random attribute selection
- Ferns
  - Fern structure
  - Semi-Naïve Bayes combination
  - Applications
Recap: Random Forests (Breiman 2001)

- General ensemble method
  - Idea: Create ensemble of many (very simple) trees.

- Empirically very good results
  - Often as good as SVMs (and sometimes better)!
  - Often as good as Boosting (and sometimes better)!

- Standard decision trees: main effort on finding good split
  - Random Forests trees put very little effort in this.
  - CART algorithm with Gini coefficient, no pruning.
  - Each split is only made based on a random subset of the available attributes.
  - Trees are grown fully (important!).

- Main secret
  - Injecting the “right kind of randomness”.

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Random Forests - Algorithmic Goals

• Create many trees (50 - 1,000)

• Inject randomness into trees such that
  - Each tree has maximal strength
    - I.e. a fairly good model on its own
  - Each tree has minimum correlation with the other trees.
    - I.e. the errors tend to cancel out.

• Ensemble of trees votes for final result
  - Simple majority vote for category.

  - Alternative (Friedman)
    - Optimally reweight the trees via regularized regression (lasso).
Random Forests - Injecting Randomness (1)

- **Bootstrap sampling process**
  - Select a training set by choosing $N$ times with replacement from all $N$ available training examples.
  - On average, each tree is grown on only $\sim 63\%$ of the original training data.
  - Remaining $37\%$ “out-of-bag” (OOB) data used for validation.
    - Provides ongoing assessment of model performance in the current tree.
    - Allows fitting to small data sets without explicitly holding back any data for testing.
    - Error estimate is unbiased and behaves as if we had an independent test sample of the same size as the training sample.
Random Forests - Injecting Randomness (2)

- **Random attribute selection**
  - For each node, randomly choose subset of $K$ attributes on which the split is based (typically $K = \sqrt{N_f}$).
  - Faster training procedure
    - Need to test only few attributes.
  - Minimizes inter-tree dependence
    - Reduce correlation between different trees.

- **Each tree is grown to maximal size and is left unpruned**
  - Trees are deliberately overfit
  - Become some form of nearest-neighbor predictor.
A Graphical Interpretation

Different trees induce different partitions on the data.

Slide credit: Vincent Lepetit
A Graphical Interpretation

Different trees induce different partitions on the data.
Different trees induce different partitions on the data.

By combining them, we obtain a finer subdivision of the feature space...
A Graphical Interpretation

Different trees induce different partitions on the data.

By combining them, we obtain a finer subdivision of the feature space...

...which at the same time also better reflects the uncertainty due to the bootstrapped sampling.
Summary: Random Forests

• Properties
  - Very simple algorithm.
  - Resistant to overfitting - generalizes well to new data.
  - Faster training.
  - Extensions available for clustering, distance learning, etc.

• Limitations
  - Memory consumption
    - Decision tree construction uses much more memory.
  - Well-suited for problems with little training data
    - Little performance gain when training data is really large.
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  - Fern structure
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  - Applications
A Case Study in Deconstructivism...

• What we’ve done so far
  - Take the original decision tree idea.
  - Throw out all the complicated bits (pruning, etc.).
  - Learn on random subset of training data (bootstrapping/bagging).
  - Select splits based on random choice of candidate queries.
    - So as to maximize information gain.
    - Complexity: \( O(KN^2 \log N) \)
  ⇒ Ensemble of weaker classifiers.

• How can we further simplify that?
  - Main effort still comes from selecting the optimal split (from reduced set of options)...
  - Simply choose a random query at each node.
    - Complexity: \( O(N) \)
  ⇒ Extremely randomized decision trees

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Extremely Randomized Decision Trees

- Random queries at each node...
  - Tree gradually develops from a classifier to a flexible container structure.
  - Node queries define (randomly selected) structure.
  - Each leaf node stores posterior probabilities

- Learning
  - Patches are “dropped down” the trees.
    - Only pairwise pixel comparisons at each node.
    - Directly update posterior distributions at leaves
  ⇒ Very fast procedure, only few pixel-wise comparisons
  ⇒ No need to store the original patches!
Performance Comparison

- Results
  - Almost equal performance for random tests when a sufficient number of trees is available (and much faster to train!).

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From Trees to Ferns...

- **Observation**
  - If we select the node queries randomly anyway, what is the point of choosing different ones for each node?
  - ⇒ Keep the same query for all nodes at a certain level.
  - ⇒ This effectively enumerates all $2^M$ possible outcomes of the $M$ tree queries.
  - ⇒ Tree can be collapsed into a fern-like structure.

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What Does This Mean?

- Interpretation of the decision tree
  - We model the class conditional probabilities of a large number of binary features (the node queries).
  - Notation
    - $f_i$: Binary feature
    - $N_f$: Total number of features in the model.
    - $C_k$: Target class
  - Given $f_1, \ldots, f_{N_f}$, we want to select class $C_k$ such that
    $$k = \arg \max_k p(C_k | f_1, \ldots, f_{N_f})$$
  - Assuming a uniform prior over classes, this is the equal to
    $$k = \arg \max_k p(f_1, \ldots, f_{N_f} | C_k)$$
  - Main issue: How do we model the joint distribution?
Modeling the Joint Distribution

• Full Joint
  - Model all correlations between features
    \[ p(f_1, \ldots, f_{N_f} | C_k) \]
    \[ \Rightarrow \text{Model with } 2^{N_f} \text{ parameters, not feasible to learn.} \]

• Naïve Bayes classifier
  - Assumption: all features are independent.
    \[ p(f_1, \ldots, f_{N_f} | C_k) = \prod_{i=1}^{N_f} p(f_i | C_k) \]
    \[ \Rightarrow \text{Too simplistic, assumption does not really hold!} \]
    \[ \Rightarrow \text{Naïve Bayes model ignores correlation between features.} \]
Modeling the Joint Distribution

- Decision tree
  - Each path from the root to a leaf corresponds to a specific combination of feature outcomes, e.g.
    \[ p_{\text{leaf}_m}(C_k) = p(f_{m1} = 1, f_{m2} = 0, \ldots, f_{md} = 1 | C_k) \]
  - Those path outcomes are independent, therefore
    \[ p(f_1, \ldots, f_{N_f} | C_k) \approx \prod_{m=1}^{M} p_{\text{leaf}_m}(C_k) \]
  - But not all feature outcomes are represented here...
Modeling the Joint Distribution

- **Ferns**
  - A fern $F$ is defined as a set of $S$ binary features $\{f_l, \ldots, f_{l+S}\}$.
  - $M$: number of ferns, $N_f = S \cdot M$.
  - This represents a compromise:
    \[
    p(f_1, \ldots, f_{N_f} | C_k) \approx \prod_{j=1}^{M} p(F_j | C_k)
    \]
    \[
    = p(f_1, \ldots, f_S | C_k) \cdot p(f_{S+1}, \ldots, f_{2S} | C_k) \cdot \ldots
    \]
    - Full joint inside fern
    - Naïve Bayes between ferns

$\Rightarrow$ Model with $M \cdot 2^S$ parameters ("Semi-Naïve").
$\Rightarrow$ Flexible solution that allows complexity/performance tuning.
Modeling the Joint Distribution

• Ferns
  ➢ Ferns are thus semi-naïve Bayes classifiers.
  ➢ They assume independence between sets of features (between the ferns)...
  ➢ ...and enumerate all possible outcomes inside each set.

• Interpretation
  ➢ Combine the tests \( f_l, \ldots, f_{l+S} \) into a binary number.
  ➢ Update the “fern leaf” corresponding to that number.

| \( f_0 \) | 0 |
| \( f_1 \) | 0 |
| \( f_2 \) | 1 |

Update leaf \( 100_2 = 4 \)
Ferns - Training

The tests compare the intensities of two pixels around the keypoint:

\[
    f_i = \begin{cases} 
        1 & \text{if } I(m_{i,1}) \leq I(m_{i,2}) \\
        0 & \text{otherwise} 
    \end{cases}
\]

Invariant to lighting change by any raising function.

Posterior probabilities:

\[
P(f_1, f_2, \cdots f_n \mid C = c_j)
\]
Ferns - Training
Ferns - Training

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Ferns - Training

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Ferns - Training Results

Normalize:

\[ \sum = 1 \]
Ferns - Training Results

\[
\sum = 1
\]
Ferns - Recognition

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Performance Comparison

- Results
  - Ferns perform as well as randomized trees (but are much faster)
  - Naïve Bayes combination better than averaging posteriors.

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Keypoint Recognition in 10 Lines of Code

1: for(int i = 0; i < H; i++) P[i] = 0.;
2: for(int k = 0; k < M; k++) {
3:   int index = 0, * d = D + k * 2 * S;
4:   for(int j = 0; j < S; j++) {
5:     index <<= 1;
6:     if (*(K + d[0]) < *(K + d[1]))
7:       index++;
8:   }
9:   p = PF + k * shift2 + index * shift1;
10:  for(int i = 0; i < H; i++) P[i] += p[i];
}

- Properties
  - Very simple to implement;
  - (Almost) no parameters to tune;
  - Very fast.

Application: Keypoint Matching with Ferns
Application: Mobile Augmented Reality

Mobile Phone Augmented Reality

at
30 Frames per Second using Natural Feature Tracking
(all processing and rendering done in software)


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Practical Issues - Selecting the Tests

• For a small number of classes
  - We can try several tests.
  - Retain the best one according to some criterion.
    - E.g. entropy, Gini

• When the number of classes is large
  - Any test does a decent job.
Summary

• We started from full decision trees...
  ➢ Successively simplified the classifiers...

• ...and ended up with very simple randomized versions
  ➢ Ensemble methods: Combination of many simple classifiers
  ➢ Good overall performance
  ➢ Very fast to train and to evaluate

• Common limitations of Randomized Trees and Ferns?
  ➢ Need large amounts of training data!
    - In order to fill the many probability distributions at the leaves.
  ➢ Memory consumption!
    - Linear in the number of trees.
    - Exponential in the tree depth.
    - Linear in the number of classes (histogram at each leaf!)
References and Further Reading

• Very recent topics, not covered sufficiently well in books yet...

• The original papers for Randomized Trees

• The original paper for Random Forests:

• The papers for Ferns:
  - D. Wagner, G. Reitmayr, A. Mulloni, T. Drummond, D. Schmalstieg, *Pose Tracking from Natural Features on Mobile Phones*. In *ISMAR 2008*. 

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